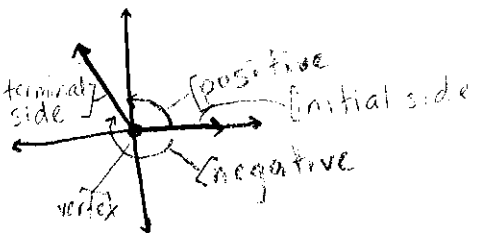
TRIG

ANGLES - BASICS

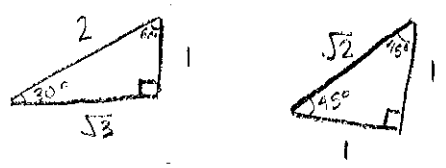


coterminal angles:
same initial and terminal sides but a different rotation
angle $+2\pi$ or $360°$

$50° + 360° = 410°$
$410°$ is coterminal to $50°$

reference angles:
acute distance to the x-axis

common right triangles
$30°\text{-}60°\text{-}90°$ $45°\text{-}45°\text{-}90°$



★ $\sin^2 x = (\sin x)^2$

INVERSES

reflected across $y = x$ · $f^{-1}(x) \cdot f^{-1}(f(x)) = x$

$\sin^{-1}(x)$
-> what angle produces "x" when plugged in

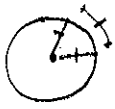
$\sin^{-1}x$ = arcsine $\neq \frac{1}{\sin x}$
$\cos^{-1}x$ = arccosine / $\frac{1}{\cos x}$
$\tan^{-1}x$ = arctangent / $\frac{1}{\tan x}$

★ standard position: $0° \rightarrow x°$

★ θ or x = typical unknown $(\sin\theta, \cos x)$

Degrees: $360°$ in a circle $\frac{1}{360} = 1°$

Radians 2π in a circle



one arclength = 1 radius
the angle created = 1 radian

CONVERSION

Degrees to radians: $A° \cdot \frac{\pi}{180°}$

Radians to degrees: $\frac{\pi}{A} \cdot \frac{180°}{\pi}$

★ $360° = 2\pi$ rad $180° = \pi$ rad. $90° = \frac{\pi}{2}$ rad.

Right Triangle Trig

sine = $\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$ cosecant = $\csc x = \frac{\text{hypotenuse}}{\text{opposite}}$

cosine = $\cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$ secant = $\sec x = \frac{\text{hypotenuse}}{\text{adjacent}}$

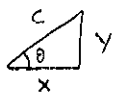
tangent = $\tan x = \frac{\text{opposite}}{\text{adjacent}}$ cotangent = $\cot x = \frac{\text{adjacent}}{\text{opposite}}$

SOH CAH TOA = memory trick

Standard form

$\frac{x}{\sqrt{a}} \rightarrow \frac{x\sqrt{a}}{a}$

THROUGH A POINT



point = (x, y)
adjacent = x
opposite = y

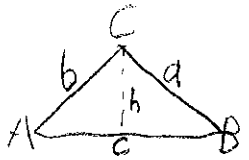
Quadrantal angle: terminal side is on an axis

$180°$: $\sin = 0$ $\csc$: DNE
$\cos = -1$ $\sec = -1$
$\tan = 0$ $\cot$: DNE

RESTRICTIONS

$\sin x$	$+\checkmark$	D: $[-1, 1]$	R: $[-\frac{\pi}{2}, \frac{\pi}{2})$
$\cos x$	$\checkmark+$	D: $[-1, 1]$	R: $[0, \pi)$
$\tan x$	$+\checkmark$	D: $(-\infty, \infty)$	R: $(-\frac{\pi}{2}, \frac{\pi}{2})$

Trig Formulas:



Laws of Sines:

$\sin A = \frac{h}{b}$ $\sin B = \frac{h}{a}$

$h = \sin A \cdot b$ $h = \sin B \cdot a$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of Cosines:

$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$b^2 = a^2 + c^2 - 2ac \cdot \cos B$

$c^2 = a^2 + b^2 - 2ab \cdot \cos C$

Reciprocal:

$\frac{1}{\sin x} = \csc x$ $\frac{1}{\csc x} = \sin x$

$\frac{1}{\cos x} = \sec x$ $\frac{1}{\sec x} = \cos x$

$\frac{1}{\tan x} = \cot x$ $\frac{1}{\cot x} = \tan x$

Quotient:

$\frac{\sin x}{\cos x} = \tan x$ $\frac{\cos x}{\sin x} = \cot x$

Even:

$\cos(-x) = \cos x$

$\sec(-x) = \sec x$

Odd:

$\sin(-x) = -\sin(x)$

$\tan(-x) = -\tan(x)$

$\csc(-x) = -\csc(x)$

$\cot(-x) = -\cot(x)$

Pythagorean:

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\cos^2 x = 1 - \sin^2 x$

$1 + \tan^2 x = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$1 = \sec^2 x - \tan^2 x$

$\cot^2 x + 1 = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

$1 = \csc^2 x - \cot^2 x$

Sum and Difference:

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Double Angles:

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

Reducing Power:

$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

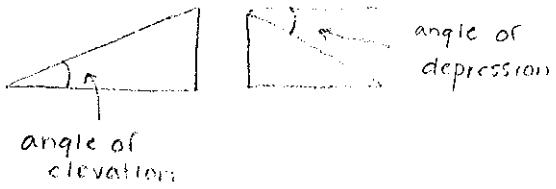
$\cos^2\theta = \frac{\cos 2\theta + 1}{2}$

$\tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$

TRIG

Applications:

4.4 - Word Problems



4.5 - Law of Sines

To Find missing side / angle:

1. Find missing angle by subtracting Known values from 180°
2. Identify known ratios
3. Plug in information and set equal to completely known ratio
4. Solve by isolating

If asked if a triangle exists:

$h = \sin A \cdot b$

when $h = a$ ⟶ right triangle
when $h > a$ ⟶ not a triangle
when $h < a$ ⟶ 2 angle options

Ambiguous Cases:

1. Set up $\sin A \cdot b$ to find h
2. Determine if $a < h$
3. If 1 or 2 triangles exist, use law of sines to find $\sin B$
4. Multiply $\sin B$ by $\sin^{-1}$ to find value
5. To determine if 2nd triangle exists, add value of $\angle B$ and its reference angle. If angles are $> 180°$ ⟶ not a triangle.

5.1 - Equations from Graphs

To write equation from graph:

1. Sine or Cosine?
2. Is it reflected?
3. Vertical shift?
4. Amplitude change?
5. Determine the period

$y = a \cdot \sin(bx) + c$

↑ amplitude ↑ function ↑ period ↑ vertical shift

6.2 - Sum + Diff Formulas

- When you see a number that you don't recognize, in parenthesis, break it down

  ex $\cos(165°)$

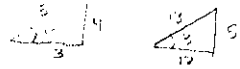
  ↓

  $\cos(120° + 45°)$

- When you see something like

  $\sin\left[\tan\frac{4}{3} - \cos\frac{12}{13}\right]$

  - Create triangles that fit with the values of the functions inside the parenthesis



  - Treat "tan" like a and "cos" like b
  - Solve

6.4 - Solving Equations

To solve:

1. Isolate the function
2. Take inverse of [illegible]
3. Determine values of x
4. Find extraneous angles [illegible]

Verifying

If you get stuck...

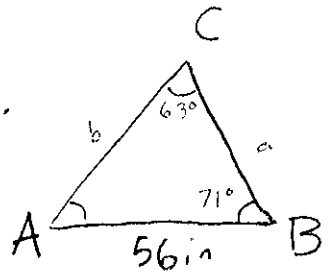
- What can you replace [illegible] expression?
- Can all terms be separated?
- Start [illegible]
- Can you combine terms?

Unit Circle Values

degree	radian	Cosine	Sine	degree	radian	Cosine	Sine
30	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	210	$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	225	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
60	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	240	$\frac{4\pi}{3}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
90	$\frac{\pi}{2}$	0	1	270	$\frac{3\pi}{2}$	0	-1
120	$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	300	$\frac{5\pi}{3}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
135	$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	315	$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
150	$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	330	$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
180	π	-1	0	360	2π	1	0

TRIG QUIZ

1.



Solve for a and b

2. $A = 30°$
$a = 10$
$b = 40$

How many triangles exist? What are they?

3. find exact value: $\tan\left(\frac{4\pi}{3} - \frac{\pi}{4}\right)$

4. find exact value: $\tan\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

5. solve $2\cos^2 x - \sin x = 1$
$[0, 2\pi)$

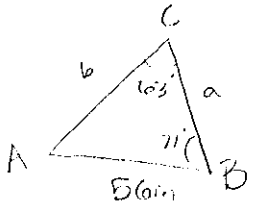
6 An A-Frame house is 35 feet wide. It's roof makes a 60° angle with the ground

- Find the length of one side of the roof, round to the nearest tenth of a foot

7. find the exact value of the expression. $\sin\left(\tan^{-1}\left(\frac{4}{3}\right) + \cos^{-1}\left(\frac{5}{13}\right)\right)$

hint
make diagrams to help your understanding

TRIG QUIZ KEY

1. 

$\angle A = 46°$
$\angle B = 71°$
$\angle C = 63°$

$\frac{c}{\sin C} = \frac{56}{\sin 63}$] known ratio

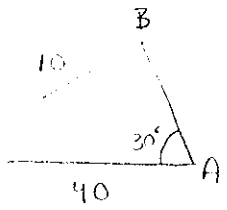
$\frac{56}{\sin 63} = \frac{a}{\sin 46}$] multiply + reduce

$a = 45.2$

$\frac{56}{\sin 63} = \frac{b}{\sin 71}$] multiply + reduce

$b = 59.43$

2. $A = 30°$
$a = 10$
$b = 40$



* [illegible]

$\sin(30) \cdot 40 = h$
$h = 20$
$10 < 20$
$a < h$
no triangle exists

3. $\tan\left(\frac{4\pi}{3} - \frac{\pi}{4}\right)$

$\frac{4\pi}{3} = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) \rightarrow \tan(\quad) = \sqrt{3}$
$\frac{\pi}{4} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \rightarrow \tan(\quad) = 1$

$\tan\alpha - \tan\beta$
$1 + \tan\alpha\tan\beta$

$\sqrt{3} - 1$
$1 + \sqrt{3}(1)$

$\sqrt{3} - 1$
$1 + \sqrt{3}$

4. $\tan\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

$\tan\left[\frac{5\pi}{6}\right]$

$\frac{5\pi}{6} = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

$\frac{1}{2} \cdot \frac{2}{\sqrt{3}}$

$-\frac{1}{\sqrt{3}}$

* make sure you put $\frac{\sin}{\cos}$ over $\frac{\cos}{\cos}$
$\sin = y$
$\cos = x$

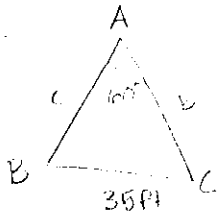
5. $2\cos^2 x - \sin x = 1 \quad [0, 2\pi)$

$2(1 - \sin^2 x) - \sin x = 1$
$2 - 2\sin^2 x - \sin x = 1$
$-2\sin^2 x - \sin x + 1 = 0$
$2\sin^2 x + \sin x - 1 = 0$
$2\sin^2 x + 2\sin x - \sin x - 1 = 0$
$2\sin x(\sin x + 1) - 1(\sin x + 1) = 0$
$(2\sin x - 1)(\sin x + 1) = 0$
$\sin x = \frac{1}{2} \quad \sin x = -1$

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

TRIG QUIZ

6.



$\angle A = 60$

$\angle B = 60$

$\angle C = 60$

* equilateral triangles' angles are all 60°!

$\frac{b}{\sin 60} = \frac{35}{\sin 60}$

$b = 35$ ft

$\frac{c}{\sin 60} = \frac{35}{\sin 60}$

$c = 35$ ft

7. $\sin(\tan^{-1}(\frac{3}{5}) + \cos^{-1}(\frac{5}{13}))$

$\sin\alpha\cos\beta + \sin\beta\cos\alpha$

$\left(\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{12}{13}\right)\left(\frac{4}{5}\right)$

$\frac{15}{65} + \frac{48}{65}$

$\frac{63}{65}$